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Quantum-inspired Genetic Algorithm with Two Search Supportive Schemes and Artificial Entanglement

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***Abstract*— In this paper, we present an enhanced quantum-inspired genetic algorithm (eQiGA) with a combination of proposed mechanisms: two search supportive schemes and artificial entanglement. This combination is aimed at balancing exploration and exploitation. Two schemes, namely Explore and Exploit scheme are designed with aggressive specific roles reflecting its name. Entanglement is considered to be one of the significant strengths in quantum computing aside the probabilistic representation and superposition. Hence we attempt to apply its concept as part of our strategy for its potential. In addition, two new sub-strategies are proposed: fitness threshold, and quantum side-stepping. The algorithm is tested on multiple numerical optimization functions, and significant results of improved performance are obtained, studied, and discussed.**

Keywords—quantum computing, genetic algorithms, search supportive scheme, numerical optimization, artificial entanglement.

I. INTRODUCTION

Quantum-inspired variant of evolutionary computation is an emerging research field attracting many researchers due to the promising strengths of quantum computers (QC). Those researchers have begun to unravel further potentials in our classical computers by studying the aspects of quantum mechanics and applying its concepts to existing evolutionary approaches. A survey by Zhang [1] explained in detail about the current trend of the quantum-inspired evolutionary algorithms (QEA) that many had claimed, tested and shown good results over conventional methods. However, it is also pointed therein that fundamental problems in heuristic search are still present even in notable works by Han and Kim [2-4]. Specifically, problems such as the premature convergence was present in [2] but apparently resolved later in [3-4], and the exploration and exploitation dilemma is still present in all available methods as it still cannot be solved but only minimized.

For the balance between exploration and exploitation, Edelkamp et al. [5] argued that a policy to ensure convergence is difficult to formulate because it remains hard for computers to answer "when is the right time to explore or exploit?." Such a policy's goal is not only just about the amount of time spent in exploring or exploiting, but also about its search behaviour whether it is well-defined or suited for the purpose of explore or exploit. In other words, those policies have a direct effect on an algorithm's performance in terms of convergence speed and stability.

In this paper, as an enhancement to quantum-inspired genetic algorithms (QiGA), we propose two supportive search schemes (TSSS), i.e. the "explore" and "exploit" schemes which are explicitly used along with two sub-strategies in order to alleviate the aforementioned problem. Those sub-strategies proposed here are the fitness-threshold (FT) strategy which is responsible for determining when to switch between the two schemes, and the quantum side-stepping (QSS) strategy for escaping local optima. In particular, these sub-strategies are the result of extensive study of QiGA's search behaviour.

Furthermore, in order to bring the benefits of entanglement in QC, we introduce another new mechanism called the artificial entanglement (AE). AE is a newly designed mechanism that conforms to a core principle of standard entanglement in QC and can also serve as a platform for more diverse strategies with the benefit of entangled solutions. Here, we aim at achieving a solution from a different angle through AE because of its correlation with the original solution. Detailed implementation of AE is described in Section IV.

In short, the contributions of this paper are summarized as follows:

1. TSSS ("explore" and "exploit" schemes) that have very specific roles, in which explore scheme offers very wide and guided search that it is able to land on a potential optimum quickly while exploit scheme is a focused search where it is able to drill down into an identified optimum;

2. AE that creates artificially entangled solutions whose conform to two core principles of entanglement: correlated values and rotational behaviour, which gives us a unique approach towards the true solution;

3. FT that serves as a bridge between the need of explore and exploit;

4. QSS that allows an efficient escape from a local optimum to a neighbour point discovered by AE.

Our eQiGA and QEA implementations in Java can be found at: http://www.ice.ci.ritsumei.ac.jp/~ruck/downloads.html

II. Previous Works

Not long after QC was first introduced by Manin [7], Feynman [8], and Deutsch [9], one of the early QiGAs was proposed by Narayanan [6]. Although at that time, his aim was apparently not to address those issues in heuristic search but rather, he was trying to create an awareness and introduce a QiGA paradigm. In his work, he demonstrated his proposed QiGA in Travelling Salesman Problem (TSP) and the results obtained surpassed conventional methods. This marked the beginning where quantum-inspired techniques began to receive attention from researchers.

A notable work by Han and Kim [2-4], who proposed the first QEA, demonstrates the applicability in another well-known problem domain called the knapsack problem (KP). In their paper, the emphasis was more on the study of characteristics of the proposed QEA. Extensive experiments were carried out with many different sets of parameters, and particularly the Q-gate or rotation gate angle setting were heavily analysed. They further suggested that because of the inherent probabilistic mechanism of QEA, it naturally leads to a good balance between exploration and exploitation. Based on the positive results of QEA, they concluded that the proposed QEA was effective and applicable in combinatorial problems. In [3-4], the QEA was tested in numerical optimization functions. As a set of improvements to the first version in [2], H_ϵ gate was introduced to tackle the issue of premature convergence by preventing a Q-bit away from 0 or 1 to a certain degree and Shannon entropy is used to investigate the exploration strategy. It was said that in [3], neither the QEA nor EP (Evolutionary Programming) algorithms were able to converge in Rosenbrock function with the dimension of 30. Latest work of Han and Kim in [4] studied the behaviour of a single individual and demonstrated some considerably good results in Rosenbrock, Step, and Shekel function on low dimensions (2, 5, and 2 respectively). However, despite the good results, clearly the exploration and exploitation issues in general still remain at large. In this respect, we aim to further tackle the issues as we believe that it strongly contributes to eQiGA's performance.

III. Basic Concepts And Insights

As a preliminary portion of this paper, this section describes the basic concepts of eQiGA and some original insights that inspired this research.

A. Representation

Similar to other proposed quantum-inspired algorithms, the smallest unit of information is stored in two-state QC and is called a quantum bit or qubit [11]. A qubit may be in state "1" or "0" or both at the same time, which is called the superposition phenomenon where, the qubit state can be given by

$$\Psi = \alpha |0\rangle + \beta |1\rangle \quad , \qquad (1)$$

where α and β are complex numbers satisfying the following constraint

$$|\alpha|^2 + |\beta|^2 = 1 \quad , \qquad (2)$$

In other words, $|\alpha|^2$ and $|\beta|^2$ give the probability that the qubit to be found in the state of 0 and 1, respectively.

In a 2D representation, the above constraint can be regarded based on a simple geometric definition where $|\alpha|^2$ is $\cos^2(\theta)$ and $|\beta|^2$ is $\sin^2(\theta)$. Hence, in an equivalent form it can also be said as

$$\cos^2(\theta) + \sin^2(\theta) = 1, \qquad (3)$$

In quantum-inspired algorithms, Han and Kim [2] introduced the term "Q-bit" for the probabilistic representation, and it is widely used across many of their other proposals instead of qubit. Hence, as follows: we also use the same definitions of Q-bit representation.

Definition 1: In eQiGA, Q-bit is defined with a pair of numbers that represent the probabilities of (α, β) in the form of

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \qquad (4)$$

where $|\alpha|^2 + |\beta|^2 = 1$. This theoretical composition of probabilistic representation allows the possibility of the bit being in the state of "1" and "0" at the same time, the phenomenon being called "superposition".

Definition 2: Each chromosome (individual) consists of an array of n Q-bits where n is the number of Q-bits predefined the user. A chromosome can be defined as

$$\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_n \\ \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_n \end{bmatrix}, \qquad (5)$$

where $|\alpha_i|^2 + |\beta_i|^2 = 1$, i = 1, 2, …, n. Theoretically speaking, this kind of probabilistic representation promotes parallelism. Consider a three-Q-bit representation of a chromosome where the possible states would be $|000\rangle$, $|001\rangle$, $|010\rangle$, $|100\rangle$, $|110\rangle$, $|101\rangle$, $|011\rangle$ and $|111\rangle$. Each of these states has a probability that the chromosome might result in any state when observed. Therefore, a change in the probabilities, also means altering many solutions at the same time. Such phenomena, theoretically allows the computational speed to be exponentially faster than classical algorithm [10]. However, such is true for real QC but in practice, this approach result in slightly higher computational time compared to simple conventional methods due to its complexity in deciphering or observing its values.

However, the concept does have its advantages; in terms of the population size, e.g. even as small as one is sufficient compared to conventional GA methods that usually have 50-100 for ensuring diversity. In a sense, lower population means lower resource consumption, which also means faster computation. Aside the conceptual advantages, it is also worthwhile to understand eQiGA's search behavior in order to further tune or manipulate it to its best.

B. Procedural Flow

Our eQiGA structure is an enhanced version of the basic QiGA proposed by Talbi et al. [10]. To complement with our proposed methods, we have modified the basic initial QiGA structure, to which is described as in Fig. 1. The procedure begins with the initialization phase where the population of chromosome(s) is initialized with random probabilities, after which they are measured and the best solution is kept as a reference for interference (or rotation) phase. As mentioned above, the algorithm only requires a small population size from 1 to 4.

The recombination phase begins with the interference operation then followed by quantum crossover where in this case, it is different than conventional method because it performs crossover by using combination-style of pairing when selecting parents. For example, if the population size is 4, then since the order is not important, the resulting number of chromosomes would be 12 offspring added with 4 initial chromosomes resulting in 16 chromosomes. Additionally, we have made a simple modification to the crossover method to change its direction so that the information of dimensions (in the case of numerical optimization) between the parents are shared instead of being drastically modified. Illustration in Fig. 2 describes the direction change. Following at the end of crossover, quantum mutation similar to implementation by Wang et al. [12] is performed. Then, quantum shift operation is performed, and lastly, measurement of the resultant chromosomes is carried out, by which, in real QC, this is supposed to 'collapse' the qubit into a single state in order to obtain a readable value. This destroys the superposition, but in our case, since this algorithm is executed using classical computers, it is the best interest that we should keep the solutions as it is for future operations [10]. After measurement, a new population will be selected based on the ranking of top fittest and the last will be randomly selected in order to maintain its diversity.

C. Rotation Gate (Quantum Interference)

A quantum unitary operator which is the rotation gate is used in the interference phase as follows:

$$U(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \qquad (6)$$

where θ is the rotation angle and its direction is determined based on the corresponding bit value of the best solution so far and the current values of the Q-bit. For determination of the rotation's direction, we adopted a simple lookup table from Talbi et al.[10]. An example of interference is shown in Fig. 3.

D. Entanglement

Quantum entanglement in QC occurs when one qubit is entangled with another qubit. It was first called as an Einstein, Podolsky, Rosen (EPR) paradox [13] for which Erwin Schrödinger then coined the term "entanglement" to describe the correlations between the particles [14]. Based on the quantum theory by Albert Einstein, the change that caused by

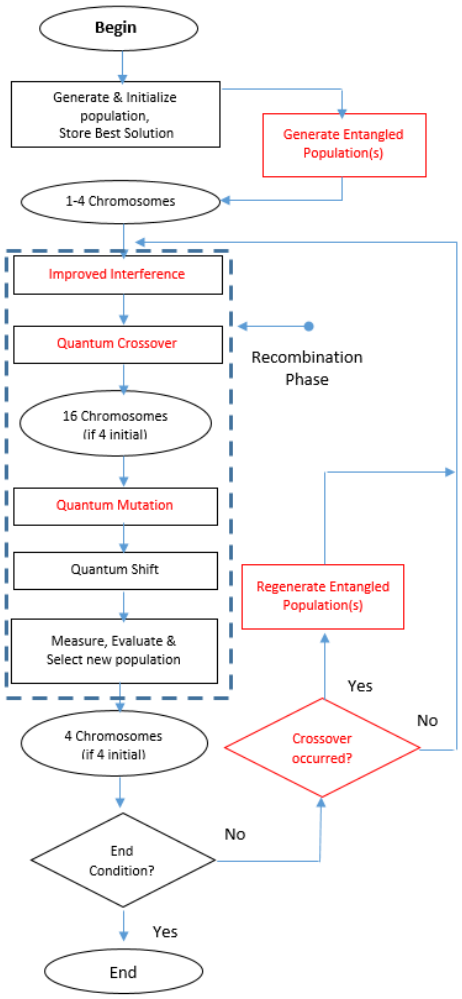

Fig. 1. Enhanced Structure of eQiGA with Artificial Entanglement, red texts indicate modified and added processes

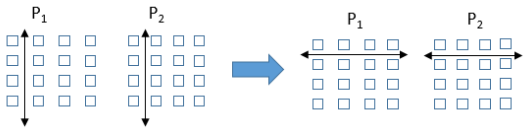

Fig. 2. Modified direction of Quantum Crossover based on the original concept by Talbi et al. [10]

entanglement is faster than the speed of light. Such is true because of its principle that when a change happens to a qubit, entangled qubits will undergo immediate change as well. Although this phenomena is impossible to accurately replicate or implement in classical computers, we exploited the concept and apply it as part of our strategy. We call it the artificial entanglement (AE) which is discussed in Section IV.

IV. Methodologies

In this paper, we present a combination of two new main strategies and two new sub-strategies. They are described in the following.

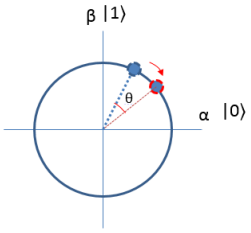

Fig. 3. Example of a Q-bit being interfered / rotated

A. Two Supportive Schemes defined, Explore and Exploit

There are many other proposed algorithms suggesting adaptive strategies for their parameters such as IQGA in [12] where the rotation angle is adaptive. However, we find that rotation angle alone is not enough to define the behavior of the search, and the amount of time spent on adapting may be unnecessary if adaption cannot be achieved within very short time. Therefore, we proposed the schemes in Table I.

Referring to Table I, we have now defined the "Local Best Solution" (LBS) and "Global Best Solution" (GBS). LBS refers to the best fittest among the chromosomes in each generation while GBS refers to the overall best fittest so far. The schemes are tested and performed very well up to their fundamental purposes which are to explicitly explore and exploit. Exploration scheme was able to search vastly for a point close to a local or global optimum quickly and exploitation scheme basically helps it to dive down into the optimum (local or global) effectively. Although we have two effective schemes that have explicit search behaviour, but there is also a need for a mechanism to determine how and when to switch between the two schemes. To address this, we further propose the Fitness Threshold which is discussed in Section IV (C).

TABLE I
SEARCH SUPPORTIVE SCHEMES FOR EXPLORE AND EXPLOIT

	Explore	**Exploit**
Angle	π / 9	π / 180
Interference based on	**Local** Best Solution	**Global** Best Solution
Shift Rate	0.5	0.2

B. Artificial Entanglement (AE)

We designed AE based on a principle of entanglement that entanglement is a physical phenomenon that occurs when pairs or groups of particles are generated or interact in ways such that quantum state of each particle cannot be described independently. Any physical changes, such as spins or measurements performed on entangled particles are found to be correlated. For example, if a pair of particles is generated in such a way that their total spin is known to be zero if a clock-wise rotation is performed on a particle, the entangled particle will found to be rotating counter-clockwise. Based on this characteristic, we apply the clockwise and counter-clockwise rotational behavior to define the correlations of our entangled Q-bits. Such behaviour is depicted in Fig. 4.

To achieve this, we begin by creating n number of artificially entangled populations where each Q-bits are entangled in the same position as in the quantum genes. The generated entangled populations have to be related to the original, by which we have the freedom to formulate an approach on how is it related to the original. In our case, we chose to start with simple method by flipping the amplitudes between α and β from second Q-bit onwards because for example, considering if we use 25 Q-bits size, the first Q-bit will act as the sign for +ve or -ve. This process coincides with "Generate Entangled Population(s)" in Fig. 1.

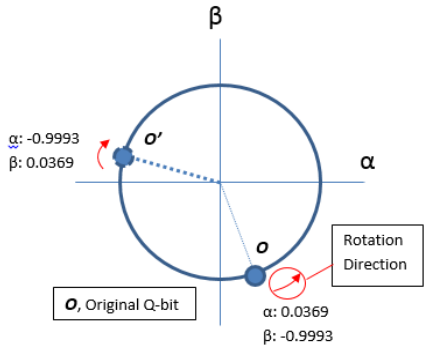

Fig. 4. Entangled Q-bit being initialized with flipped probabilities and example of rotational behaviour

To preserve the integrity of the entangled populations, no other operations such as quantum crossovers, quantum mutations and quantum shifts are allowed to be applied upon them but only the after-effect of the quantum interference which is done on the original population. Furthermore, in order to make the interference more meaningful on the entangled populations because there is no point if they are rotating at the same angle even if they are on different quadrant, therefore we introduce "reversed rotational angle sensitivity" to de-amplify the original interference angle acted upon them as the original values are considered rather large. In this case, the rotational angle for entangled populations are defined as:

$$\theta_{e_i} = \frac{\theta}{i}, \tag{7}$$

where i is the i^{th} entangled population. Quantum interference on entangled populations are performed in addition to existing quantum interference hence we described it as "Improved Interference" in Fig. 1. To optimize the functionality of this mechanism, we suggest additional two parameters: $P_{measureEntangled}$ for probability to observe the entangled solutions and $P_{swapEntangled}$ for swapping the original chromosome with entangled chromosome if it is found to be a better GBS during measurement phase. Additionally, AE is also used to discover reliable QSS points wherein upon measurement it checks with another parameter $D_{sideStepThreshold}$ that if it is within the threshold

then add into the *pointList* (See Algorithm 1). As of now, there are two major roles for AE which are: finding potentially better GBS with a chance to swap and discovering reliable QSS points.

C. Fitness Threshold (FT)

As we mainly testing on numerical optimization functions, we manually defined a fitness threshold for which to tell the eQiGA when to switch to exploitation. In other words, once the evaluated fitness is found to be within the threshold, it will immediately switch to exploitation and "dive" into the assumed optimum. A simple illustration in Fig. 5 describes the example behaviour of FT.

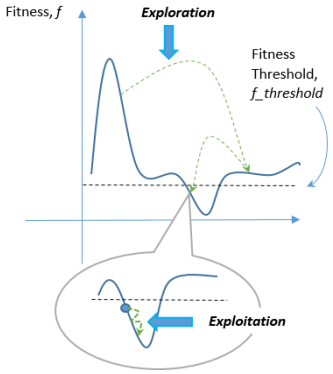

Fig. 5. An example describing how the algorithm would change its scheme

Although, we do not expect this method to be fool-proof as this approach is only effective if the "objective" is known. If it were to search an unknown space where the "objective" is not known, it may become difficult to converge. Hence, we are seeking a better mechanism that is able to determine the point of change by itself of which the theoretical implementation is discussed when we conclude this paper in Section VI.

D. Quantum Side-Stepping (QSS)

There was another issue that had been evidently present in eQiGA based on our initial tests and QEA in [3] is that it tends to get trapped in a local optimum or due to sensitivity of a function such as the Rosenbrock function as explained by Han and Kim [3] and unable to escape. To resolve this, we proposed the QSS. On contrary to back-tracking or back-stepping, we called this approach as side-stepping because of its characteristics where it does not fall back to a point based on history of the search but instead, it diverts the search focus to a proximity point by directly adapting the GBS to the neighbour solution. It then, guided by the newly adapted GBS, it will approach towards convergence "from a different direction".

Proximity points are discovered using the mechanisms of AE because based on experiments, we find that the points are more reliable for a "different direction" and it is very unlikely to return to the same trap as compared to trace-back points. The QSS only occurs during Exploit scheme due to its narrow and focused search ability, therefore unable to escape. Once QSS occurs, eQiGA will switch back to Explore scheme if it is outside of the FT. To keep the discovered points, we maintain a small list, *pointList* that holds up to ten points which are validated as the GBS gets updated for its validity in distance. Procedure of QSS can be seen in Algorithm 1.

Algorithm 1: quantumSidestep(*pointList*)

```
/*pointList is populated by AE*/
/*trapped_cost derived from number of function calls while
GBS remained the same */
if globalBestUpdate is false then
   trapped_cost ← add 1
else
   trapped_cost ← reset trapped cost back to 0
end if

/*validate pointList to ensure the points are within distance
 with GBS*/
For i = size of pointList to 0
   if fitness of pointList(i) - fitness of GBS >
                  d_sideStepThreshold then
         remove pointList(i)
   end if
end for

if trapped_cost > trapped_threshold then
   if size of pointList > 0 then
      globalBest ← get a random point from pointList
   end if
   trapped_cost ← reset trapped cost back to 0
end if
return
```

V. Performance Evaluation And Discussion

In evaluating our proposed algorithm, we performed extensive experiments to verify its performance, and put it on comparison with Han and Kim's QEA [3-4] as we had successfully implemented it and reproduced similar reported results. For the ease of reference, we use the same comparison units as in their work which are the search cost, number of times of the test functions being called in each trial, and the mean (m.), standard deviation (σ) and success rate (r.) are collected. We used the following numerical optimization functions in our tests:

Rosenbrock Function (De Jong 2, f_{Ros}): Minimize

$$f(x) = \sum_{i=1}^{N-1}(100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2), \quad (8)$$

where $-2.048 \le x_i \le 2.048$. The global minimum value is 0.0 at $(x_1, x_2, x_3, x_4 \ldots x_N) = (1, 1, 1, 1 \ldots 1)$.

Step Function (De Jong 3, f_{Step}): Minimize

$$f(x) = \sum_{i=1}^{n} integer(x_i), \quad (9)$$

where $-5.12 \le x_i \le 5.12$. The global minimum value is at -5 multiplied with n dimensions for all $-5.12 \le x_i < -5.0$.

Shekel Function (De Jong 5, f_{Shekel}): Minimize

$$f(x) = \frac{1}{\frac{1}{k} + \sum_{j=1}^{25} g_j^{-1}(x_1,\ x_2)}, \qquad (10)$$

where $g_j(x_1, x_2) = c_j + \sum_{i=1}^{2}(x_i - a_{ij})^6$, $-65.536 \le x_i \le 65.536$, $K = 500$, $c_j = j$, and $[a_{ij}]$ is

$$\begin{bmatrix} -32 & -16 & 0 & 16 & 32 & -32 & -16 & \cdots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & -16 & \cdots & 32 & 32 & 32 \end{bmatrix}$$

The global minimum is 0.998 at $(x_1, x_2) = (-32, -32)$.

For fairest possible comparisons, we set both our proposed eQiGA and QEA domain variables (refer Table II) to be the same, such as the Q-bits size is 25 where the first Q-bit is the sign and the maximum number of trials are 100, and 50 respectively. As for the termination condition, we adopted value-to-reach (VTR) [15] by which the algorithm terminates once the fitness reaches 1e-6. However, given the circumstance that each function's global minimum may not necessary be $f = 0$, the initial VTR of 1e-6 can be applied to many functions such as Sphere, Rastrigin or ill-scaled Rosenbrock in general but in our case, it is adopted accordingly for f_{Step} and f_{Shekel}.

TABLE II
DOMAIN (PROBLEM) VARIABLES

Q-Bits Size	25
Dimension, *N*	30 (2 for f_{Shekel})
Termination Condition, VTR (OR Search Cost > 2,000,000)	Rosenbrock, $f < 1e\text{-}6$ Step, $f = -150.0$ Shekel, $f < 0.9986$

As for algorithm-specific parameters, Table III and Table IV defines the values which we used for our experiments. The set of parameter values are empirically chosen as a result of fine tuning where they reflect the best performance by far of both algorithms based on preliminary testing cases. Preliminary tests for QEA include parameters proposed by Han and Kim in [3] and [4]. Interestingly, the ideal population size of both algorithms is the same which is 2. For eQiGA, it is significant that it is more than 1 because of Quantum Crossover advantage.

TABLE III
EQIGA PARAMETERS VALUES

Quantum Crossover Rate	70%
Quantum Mutation Rate	30%
Quantum Mutation Threshold	5%
Quantum Shift Rate	Based on TSSS (Table I)
Rotation Angle, θ	Based on TSSS (Table I)
# of Measure	1
# of Entangled Populations	3
$P_{measureEntangled}$	1%
$P_{swapEntangled}$	50%
$D_{sideStepThreshold}$	25.0
$V_{trappedTheshold}$	5000
Fitness threshold (Problem-Dependent)	Rosenbrock: 2.0 Step: -135.0 Shekel: 1.5

TABLE IV
QEA PARAMETERS VALUES

Global migration period	100
Local group size	2
ε for H_ε gate	0.01
# of Measure	1
Rotation Angle, θ	0.04π

Gray coding was used for QEA, but not for eQiGA.

Table V shows the result of the experiments where direct comparisons are made. Based on the results shown, at a glance, we can see that our proposed approach gave promising results over QEA: a very significant achievement for f_{Ros} and f_{Step} as previously, it was reported by Han and Kim [3] that no algorithm was able to solve f_{Ros} with $N = 30$, and 57% improvement in terms of mean for f_{Shekel}. In the case of QEA, it is still unable to converge in f_{Ros} after 2,000,000 evaluations. Analyzing deeper into the overall results of eQiGA, we noticed that the standard deviation values are considerably high. This is because in certain trials of eQiGA, the algorithm was able to solve the problem very quickly while in some required longer time to reach VTR.

We first thought if this high standard deviation issue could be due to QSS, which is why we separated the trials to investigate; however that does not seem to be the case. Based on the QSS occurred and No-QSS occurred results of f_{Ros} in Table V, it appears to be stable although the results of QSS occurred shown to be higher than cases of No-QSS, it is only natural because of the QSS mechanism. However, looking at that the values in No-QSS occurred cases indicate that the proposed schemes are indeed effective in solving the problem within short time. Also, if we consider the cases in f_{Step} and f_{Shekel}, QSS did not trigger at all and the standard deviation is still high. We believe that it may be due to the FT mechanism that serves as a bridge to switch between the two schemes by which we aim to improve with a smarter mechanism.

In a minor comparison with CEP [16] variant such as Simulated Annealing (SA), eQiGA appeared to be converging slightly slower on simple problems such as the Shekel function (sample results with same dimension can be observed in [4]). This could be explained that since the proposed method has the characteristics of aggressive exploration, it has a possibility of overstepping some potential solutions nearby. SA is considered as a CEP [16] which is known for good at searching small local neighbourhood. However, even so, compared with the same FEP [16] variant, eQiGA still managed to achieve better results, which is worth further investigating of its behaviour.

In verifying eQiGA's search behavior, based on the convergence of eQiGA in Fig. 6, it is clear that it begins with an aggressive exploration and then followed by intense exploitation. Aggressive exploration that is guided by LBS limits the randomness of the search, which in a sense, it scans the area around the LBS rather than giving completely random solutions. On contrary to conventional GA, with no elitism is employed, LBS changes drastically in every generations, as compared to GBS that remains the same until a better solution is found. The reasons why we differentiate the focus of the quantum interference of which solution to use as a lead example

TABLE V
EXPERIMENTAL RESULTS OF EQIGA FOR TEST FUNCTIONS (8) – (10) COMPARED WITH RESULTS OF HAN AND KIM'S QEA

		m.	σ	r.	m. ending fitness
f_{Ros} (De Jong 2)	eQiGA	16637.2	12161.2	100/100	4.2E-07
	QEA	-	-	0/100	1.894
f_{Step} (De Jong 3)	eQiGA	2240.1	1749.5	100/100	-150
	QEA	61413.6	54865.0	100/100	-150
f_{Shekel} (De Jong 5)	eQiGA	2564.5	1719.0	100/100	0.9983
	QEA	5944.5	10511.5	100/100	0.9982

eQiGA f_{Ros} result breakdown	m.	σ	r.
QSS occurred	27597.1	12273.4	39/39
No-QSS occurred	9630.0	4541.6	61/61

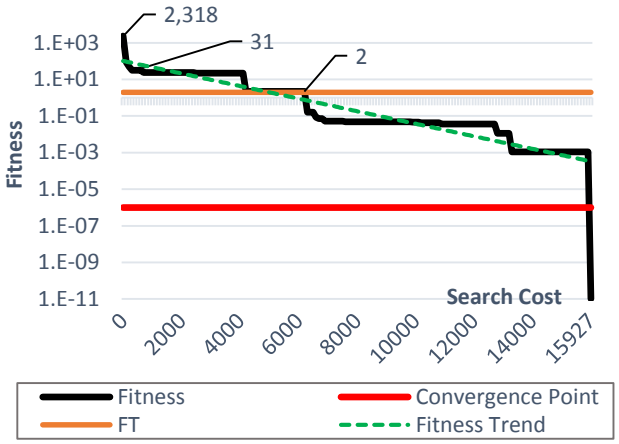

Fig. 6 A convergence sample of a single trial in eQiGA with fitness recorded every 100th search cost in f_{Ros}

for the intended distinct behaviour of explore and exploit are simply to encourage better diversity of the approach towards the true solution and avoid cases of premature convergence.

Additionally, $P_{measureEntangled}$ and $P_{swapEntangled}$ are carefully selected from extensive experiments with different probability rates. To our findings, entangled chromosomes do occasionally produce better solutions. However, swapping with high frequency has a high tendency of landing in a local optimum based on our experiments while occasional swaps encourages faster convergence in general as compared to experiments without swapping at all. In other words, even if the swaps are only occasional, it is still able to make a good leap towards the global optimum. The act of measuring the entangled chromosomes has a slight trade-off for performance as it consumes search cost to evaluate the measured solutions. Therefore, since there is no need for frequent swapping, we chose a very low probability rate to trigger the measurement of the entangled chromosomes and still maintain the exclusive benefits of the AE.

VI. CONCLUSION AND FUTURE WORK

This paper presents a novel set of methods that are proven to outperform QEA based on the results obtained. Although, there are still remaining issues that we wish to address such as the better solution for FT so that it is able to determine the strategy change by itself and reduce number of user-controlled parameters. AE is a potential platform for better improvements as it still has many aspects that can be manipulated. In particular, the results have demonstrated the effectiveness of the proposed methods in terms of convergence speed and certainly further alleviated the fundamental problems of heuristic search mentioned in Section I.

REFERENCES

[1] Zhang, G., "Quantum-inspired evolutionary algorithms: a survey and empirical study." Journal of Heuristics 17.3 (2011): 303-351.

[2] Han, K.H., Kim, J.H., "Quantum-inspired Evolutionary Algorithm for a Class of Combinatorial Optimization," IEEE Transactions on Evolutionary Computation, Piscataway, NJ: IEEE Press, vol. 6, no. 6, pp. 580-593, Dec. 2002.

[3] Han, K.H., Kim, J.H., "Quantum-inspired Evolutionary Algorithms with a New Termination Criterion, H_ε Gate, and Two-Phase Scheme," IEEE Transactions on Evolutionary Computation, Piscataway, NJ: IEEE Press, vol. 8, no. 2, pp. 156-169, Apr. 2004.

[4] Han, K.H., Kim, J.H., "On the analysis of the quantum-inspired evolutionary algorithm with a single individual." Evolutionary Computation, 2006. CEC 2006. IEEE Congress on. IEEE, 2006.

[5] Edelkamp, S., Stefan S., "Heuristic search: theory and applications." Pg. 542. Elsevier, 2011.

[6] Narayanan, A., Mark M., "Quantum-inspired genetic algorithms." Evolutionary Computation, 1996, Proceedings of IEEE International Conference on. IEEE, 1996.

[7] Manin, Y. I., "Vychislimoe i nevychislimoe" [Computable and Noncomputable] (in Russian). Sov.Radio. pp. 13–15, 1980.

[8] Feynman, R. P., "Simulating physics with computers". International Journal of Theoretical Physics 21 (6): 467–488. doi:10.1007/BF02650179, 1982.

[9] Deutsch, D. (1992), "Quantum computation". Physics World

[10] Talbi, H., Amer D., and Mohamed B., "A new quantum-inspired genetic algorithm for solving the travelling salesman problem." Industrial Technology, 2004. IEEE ICIT'04. 2004 IEEE International Conference on. Vol. 3. IEEE, 2004.

[11] T. Hey, "Quantum computing: An introduction," in Computing & Control Engineering Journal. Piscataway, NJ: IEEE Press, June 1999, vol. 10, no. 3, pp. 105–112, 1999.

[12] Wang, R., et al. "An improved quantum genetic algorithm with mutation and its application to 0-1 knapsack problem." Measurement, Information and Control (MIC), International Conference on. Vol. 1. IEEE, 2012.

[13] Einstein A, Podolsky B, Rosen N., "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?". Phys. Rev. 47 (10): 777–780, 1935.

[14] Kumar, M., Quantum, Icon Books, 2009, p. 313.

[15] Storn, R., and Kenneth P.. "Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces." Journal of global optimization 11.4 (1997): 341-359, 1997

[16] Xin, Y., Liu, Y., and Lin, G.. "Evolutionary programming made faster." Evolutionary Computation, IEEE Transactions on 3.2 (1999): 82-102, 1999.